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EFFECT OF EVAPORATION ON THE DYNAMICS OF FALLING DROPS

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The relationship between the dimensionless intensity of the evaporation of a drop and the Reynolds number of the separating vapor is shown. Conditions are indicated under which the Dukovicz correction to the resistance coefficient of a falling evaporating drop cannot be ignored.

Evaporating and condensing drops find wide application in heat exchangers and other technological apparatus; therefore, their motion has been investigated theoretically by many authors [1-7]. In [6, 7] and other works, the effect of Knudsen numbers on the dynamics of a flying drop moving in its own vapor was investigated, with the temperature of the drop and intensity of evaporation being determined by the process itself. It is shown in [5] that a decrease in the size of a particle exerts a substantial influence on its motion. However, in [5] the change in the resistance coefficient of the particle due to evaporation was not taken into account. In what follows, the motion of a falling drop is considered with allowance for the effect of vapor mass injection from the boundary layer of the drop on its resistance coefficient in the regime of continuous medium. Here, we shall restrict ourselves to the case where the drop that reached the saturation temperature occurs in a medium with a markedly higher temperature and smaller content of its vapor. Then, in contrast to [6, 7], the value of N can be assumed constant (the external parameter independent of the motion and evaporation of the drop).

If in the process of motion the particle loses its substance (for example, during evaporation), then it is a body of variable composition. We will assume for simplicity that the drop moves in a gas whose properties are close to those of the drop vapor.

As is known, the motion of a material point of variable composition (as well as of a body in translational motion) is described by the Meshcherskii equation:

$$m d\mathbf{V}/dt = \Sigma \mathbf{F} + \mathbf{P}\,, \quad \mathbf{P} = \mathbf{U}_v dm/dt\,, \quad \mathbf{U}_v = \mathbf{V}_{v.a} - \mathbf{V}\,, \tag{1}$$

Obviously, if the substance separates from the drop in all directions with the same (relative to the drop) velocity and intensity, then $\mathbf{U}_v = 0$, and the reactive force reduces to zero, $\mathbf{P} = 0$.

Just as in [3-5], of the external forces $\mathbf{F}$ that are exerted on a spherical drop falling freely in a quiescent gas, we take into consideration the gravity and resistance forces:

$$m dV/dt = - mg - 0.5 C \rho V \,|V|\, \pi \delta^2/4\,. \tag{2}$$

In the present work, we consider the motion of Stokesian drops whose resistance coefficient at a constant mass is calculated from the formula $C_S = 24/\mathrm{Re}$. If the stabilization time of the boundary layer of the particle is much smaller than the characteristic times of the change in δ and V, then the process of particle motion can be regarded as quasistationary, and we may use the correction to the resistance coefficient for the effect of evaporation: $C = kC_S$. When a moving drop evaporates, its mass is injected into its boundary layer, leading to a decrease in C and to $k < 1$. The coefficient k can be calculated from Dukowicz's formula [1] ($\mathrm{Re}_v = V_v\delta/\nu_v > 0$, where V_v is the mass velocity of separating vapor relative to the drop):

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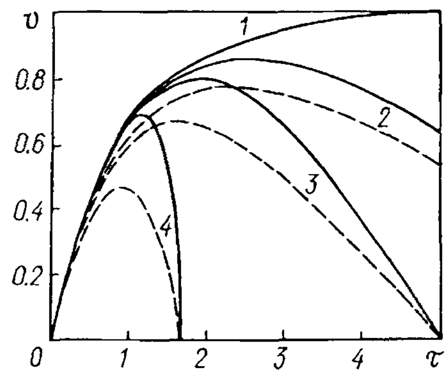

Fig. 1. Change in the velocity of a freely falling drop at $\lambda = 1$ and different values of the dimensionless intensity of evaporation: 1) $n = 0$; 2) 0.1; 3) 0.2; 4) 0.6; dashed curves, calculation without account for the correction k..

$$k = 2/3\,\mathrm{Re_v} f/(0.5\mathrm{Re_v^2} - f)\,,\quad f = 1 - (1 + \mathrm{Re_v})\exp(-\mathrm{Re_v}) \tag{3}$$

or, at very small $\mathrm{Re_v}$ values, from the formula [2]

$$k = 1 - 7/24\mathrm{Re_v}\,. \tag{4}$$

Formulas (3) and (4) remain valid also for condensation of vapor on drops [1, 2], but then the mass seems to be sucked from the drop boundary layer, $\mathrm{Re_v} < 0$ and $k > 1$.

When the drop is evaporated with a constant intensity N, the change in the square of its diameter is described by the well-known square-law dependence on time:

$$(\delta/\delta_0)^2 = 1 - Nt = 1 - n\tau\,. \tag{5}$$

Dividing both sides of Eq. (2) by mg with allowance for Eq. (5), we obtain a dimensionless equation for the motion of a variable-mass drop falling under nonvariable conditions in a quiescent medium:

$$dv/d\tau + kv/(1 - n\tau) = -1\,, \tag{6}$$

where the nondimensionalization scales are θ_0, which is the relaxation time of the drop at the initial time instant, and $W_0 = g\theta_0$, which is the levitation velocity of the drop with initial diameter δ_0.

Expression (6) is a linear inhomogeneous differential equation having an analytical solution, which under zero initial conditions ($v_0 = 0$, $x_0 = 0$) has the following form:

$$u = \begin{cases} \left[1 - (1 - n\tau)^{\frac{k-n}{n}}\right](1 - n\tau)/(n - k)\,, & k \neq n\,, \\ (1/n - \tau)\ln(1 - n\tau)\,, & k = n\,. \end{cases} \tag{7}$$

Integrating Eq. (7) with respect to time, we obtain a relation for the coordinates of the drop:

$$x = \begin{cases} \dfrac{1}{n - k}\left[\tau - \dfrac{n\tau^2}{2} + \dfrac{(1 - n\tau)^{\frac{k+n}{n}} - 1}{n + k}\right], & k \neq n\,, \\ \left(\dfrac{\tau}{n} - \dfrac{\tau^2}{2} - \dfrac{1}{2n^2}\right)\ln(1 - n\tau) + \dfrac{\tau^3}{6} - \dfrac{\tau}{2n}, & k = n\,. \end{cases} \tag{8}$$

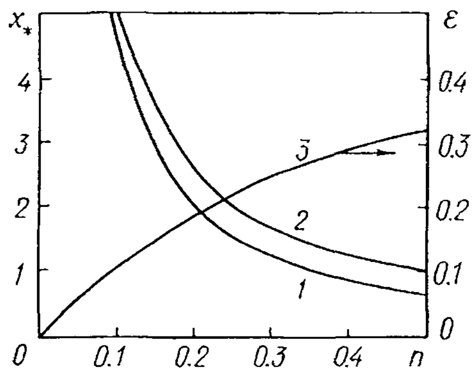

Fig. 2. Dimensionless distance x_* of the complete evaporation of a drop: 1) with account for the correction k, 2) without its account, 3) the error ε in calculation of x_* without account for the correction k.

In many works dealing with calculation of the motion of evaporating drops (for example, [3-5]), it is assumed that $k = 1$, i.e., that $\mathrm{Re_v} \to 0$ even when the values of n are appreciable. We will show that this is inadmissible because of the relationship between n and $\mathrm{Re_v}$. From Eq. (5) it follows that

$$dm/dt = -0.25\pi N\delta\rho\,. \tag{9}$$

The velocity of the separating vapor can be found from its mass flux $V_v = G_v/\rho_v$, but $G_v\pi\delta^2 = -dm/dt$, whence, according to Eq. (9),

$$V_v = 0.25N\,(\delta_0^2/\delta)\,(\rho/\rho_v)\,. \tag{10}$$

Then, the Reynolds number of the separating vapor is $\mathrm{Re_v} = 4.5\lambda n$, where $\lambda = \rho_g/\rho_v$ is the ratio of the gas and vapor densities.

The dependences of the falling-drop velocity on time at various dimensionless intensities of evaporation are presented in Fig. 1. The dashed curves show the results of calculation without account for the effect of mass injection into the boundary layer on the resistance coefficient of the drop. Due to a decrease in the value of k in evaporation, the maximum of the drop velocity increases.

The distance x_*, over which the drop evaporates completely, is very important for applications. The dimensionless time of complete evaporation at $n = \mathrm{const}$ is equal to $\tau_* = 1/n$ irrespective of the account for $k(n)$. However, because of the change in the falling-drop velocity, it is necessary to account for the function $k(n)$ in order to determine x_* correctly. The dependence of the dimensionless distance of complete evaporation of a droplet on the intensity of evaporation, obtained on substitution of τ_* into formula (8), is shown in Fig. 2. This figure also contains a dependence for the error in the calculation of x_*:

$$\varepsilon = (x_{*1} - x_*)/x_*\,.$$

If the function $k(n)$ is not taken into account, the error of the calculation of x_* increases with an increase in n; when $n = 0.04$, the error exceeds 5%, while at $n = 0.5$ the value of ε is already over 30%.

NOTATION

m, δ, ρ, mass, diameter and density of drop; ρ_v, ρ_g, densities of vapor and gas; $\lambda = \rho_g/\rho_v$; $\theta = \delta^2\rho\,/\,(18\nu_g\rho_g)$, relaxation time of drop; ν_g, kinematic viscosity of gas; C, resistance coefficient; g, free-fall acceleration; $V_{v.a}$, absolute velocity of separating vapor; U_v, relative velocity of vapor; P, reactive force; $n = N\theta_0$, dimensionless intensity of drop evaporation; $\tau = t/\theta_0$, dimensionless time; W_0, velocity of drop levitation; V, v, dimensional and dimensionless velocities of drop, $v = V/W_0$; G_v, mass flux of vapor; k, dimensionless quantity indicating a change in the resistance coefficient of drop due to evaporation; $L_0 = W_0\theta_0$, relaxation length of a drop of diameter δ_0; x,

X, dimensional and dimensionless coordinates of drop, $x = X/L_0$; x_*, x_{*1}, dimensionless distance of complete evaporation of drop calculated with account for a change in k and without it, respectively; τ_*, dimensionless time of complete evaporation of drop; $Re = V\delta/\nu_g$, Reynolds number of drop; $Re_v = V_v\delta/\nu_v$, Reynolds number of vapor; $V_v = G_v/S$, S, surface area of drop; ε, error of calculation of x_*. Subscripts: a, absolute velocity of vapor; v, vapor; g, gas; 0, initial time instant ($t = 0$); *, time instant of complete evaporation of drop; S, regime of Stokesian resistance.

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